Spatiotemporal Antiphase Dynamics in Coupled Extended Optical Media

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Experimental evidence of spatiotemporal antiphase dynamics is given for an extended system made of two liquid crystal slices that are optically coupled by two equal amplitude counterpropagating pumping beams. Theory and experiments carried out in a transverse one-dimensional configuration show that roll patterns are generated in each slice. These rolls are spatially in-phase or antiphase for a focusing or a defocusing nonlinearity type, respectively. These in-phase or antiphase dynamics remain robust even for complex spatiotemporal regimes such as dislocation regimes.

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Antiphase dynamics is a general and very well-known phenomenon which spans many fields of science. When it occurs, two quantities exhibit the same evolution versus time or space but shifted by half a period. The most well known is the temporal antiphase between two different variables \( V_1(t) \) and \( V_2(t) \) corresponding to two coupled systems or subsystems. Temporal antiphase has been demonstrated theoretically and experimentally in a large variety of science domains, from optics [1–3] to biology [4], neurology [5–7], geophysics [8], etc. On the other hand, spatiotemporal antiphase between two different variables \( V_1(r, t) \) and \( V_2(r, t) \), corresponding to two coupled spatially extended subsystems, has only been predicted theoretically [9–11]. For instance, spatial antiphase is expected to occur between, e.g., polarization components of a field ruled by the vector complex Ginzburg-Landau equation [10], periodic patterns in light counterpropagating through a system of two resonant thin films [11], or transverse localized structures in the pump and signal fields of a degenerate optical parametric oscillator with saturable absorber [9]. Antiphase dynamics in spatially extended systems has also been investigated in the case of a unique variable \( V \) recorded at two different locations of the transverse space \( V(r_1, t) \) and \( V(r_2, t) \) of a single system. This is also known in the literature as spatiotemporal antiphase dynamics [12–14] but does not deal with coupled (sub)systems. However, to the best of our knowledge, spatiotemporal antiphase in coupled (sub)systems has yet to be experimentally demonstrated.

In this Letter, we report on the experimental evidence of spatiotemporal antiphase dynamics in an optical extended system made of two coupled Kerr subsystems that are optically pumped by equal amplitude counterpropagating beams. In this system, the coupling is intrinsic since the two subsystems are part of a unique system that cannot be dissociated, whereas the coupling would have been external in the case of two initially independent systems. Here, the two different variables \( V_1 \) and \( V_2 \) correspond to the fields observed in the two subsystems. We show that stationary roll patterns in each slice are either spatially in-phase or antiphase for a focusing or a defocusing nonlinearity, respectively. These in-phase or antiphase dynamics remain robust for complex spatiotemporal regimes such as a dislocation regime [15]. Analytical developments and numerical simulations are in excellent agreement with experiments carried out for a transverse one-dimensional configuration on nematic liquid crystals slices as Kerr media.

The experimental setup follows the idea of Logvin [11] and is made of two liquid crystal (LC) layers distant from \( 2d \), and subjected to two equal amplitude \( F \) counterpropagating laser beams as shown in Fig. 1. This system is an unfolded version of the Kerr slice with simple optical feedback [16]. The light beam transmitted through one layer carries phase modulations that are converted into amplitude modulations during the free propagation over the coupling distance \( 2d \) (Talbot effect [17]) and drives the refractive index of the opposite layer via the Kerr effect. The patterns observed in both LC layers are simultaneously monitored by CCD cameras via the two transmitted beams. For the sake of simplicity, we concentrate on the case of one-dimensional extended patterns.

Apart from a different optical arrangement as shown in Fig. 1, the experimental details are identical to those reported in our previous studies on the spatiotemporal dynamics of a Kerr slice submitted to the feedback of a single mirror [15]. The coupling distance \( 2d \) governs the
nonlinearity of the system. The use of a 4f lens arrangement inserted between the two slices allows for positive as well as negative 2d values. A positive (negative) value of the coupling distance corresponds to a positive (negative) effective Kerr nonlinearity.

The global features of the scenarios of successive pattern destabilizations do not depend on the sign of 2d. For low pumping beam intensity I = |F|^2, the fields observed in the two LC slices simply reflect the overall Gaussian dependence of I. At primary threshold I_c, stationary rolls with identical wavelength for both transmitted fields are observed. As the input power is increased beyond a secondary threshold I_d, these rolls destabilize via time periodic dislocations associated with fringe annihilation (creation) for positive (negative) 2d values [15].

Figure 2 shows typical spatiotemporal evolutions of the two 1D patterns V_1 = I_1 (x) and V_2 = I_2 (x) observed in slices 1 and 2 above the primary threshold for negative [Figs. 2(a)–2(d)] and positive [Figs. 2(e)–2(h)] coupling distances 2d. These patterns V_1 and V_2 reveal in-phase or antiphase dynamics depending on the sign of the nonlinearity. Above the primary threshold, the stationary patterns formed on both beams coincide spatially [Figs. 2(e) and 2(f)] if 2d > 0, while they are shifted by half a wavelength if 2d < 0 [Figs. 2(a) and 2(b)]. We take as a definition for spatiotemporal antiphase V_1 (r, t) = V_2 (r + x/2, t), where x is the spatial wavelength of the pattern. To exhibit qualitatively these in-phase or antiphase properties, the sum (Σ) and difference (Δ) of I_1 and I_2 are constructed. The Δ pattern exhibits either deep contrast fringes [Fig. 2(d)] or almost no contrast [Fig. 2(h)] for antiphase and in-phase patterns, respectively. The Σ pattern in Fig. 2(c) has a spatial wavelength twice the wavelength of the individual patterns. This indicates that the individual patterns are in antiphase. A quantitative indication of these in-phase or antiphase dynamics has been calculated using the time averaged spatial cross correlation C_{1/2}(Δx) between the two beams I_1 (x) and I_2 (x + Δx) where Δx is the spatial shift [18]. The results which are obtained at the thresholds for pattern formation I_c and for dislocation instability I_d are presented in Table I. The values for Δx = 0 clearly identify the type of dynamics (in-phase or antiphase). The evolution of C_{1/2} versus Δx depicts damped oscillations with wavelength Λ due to the Gaussian profile of the pumping beams [18]. The maximal values of the extrema of C_{1/2}(Δx) demonstrate the very good quality for both in-phase and antiphase phenomenon since C_{1/2} is always far from 0 even within the dislocation regime. These values decrease slowly as the control parameter I_c is driven far from thresholds.

These experimental observations have been compared with the analytical predictions and the results of numerical simulations performed on an extension of the model of the single Kerr slice with feedback first introduced by Akhmanov et al. [19] and Firth and D’Alessandro [16]. The refractive indices n_{1,2} in slices 1 and 2, respectively, are ruled by the following set of coupled equations

$$\left( \tau \frac{\partial}{\partial t} - i \Delta \frac{\partial^2}{\partial x^2} + 1 \right) n_{1,2} = |F|^2 + |e^{i\alpha(x^2/\sigma^2)}e^{i\phi}|^2 + \sqrt{\varepsilon} \xi_{1,2},$$  

where F is the input Gaussian laser field with radius σ and transverse dependence F(x) = F_0 \exp(-x^2/\sigma^2). t and x are the time and transverse space variables scaled with respect to the relaxation time τ (∼2.3 s) and the diffusion length l_0 (∼10 μm). e (=0.01) scales the noise amplitude and ξ_1,2(x, t) are Gaussian stochastic processes of zero mean and delta correlation introduced as previously to model thermal noise [20]. We have set σ = d/k_0 l_0^2 where k_0 is the optical wave number of the field. The Kerr effect is parametrized by χ which is focusing (defocusing) for positive (negative) values of 2d.

For plane wave input, F(x) = F_0 and in absence of noise (e = 0), Eq. (1) admits a homogeneous stationary solution such that n_{1,2} = n_0 = 2 |F|^2. This solution destabilizes against transverse spatial modulations in the form Δn_{1,2} = a_{1,2} \exp(ikx) where K is the transverse wave number and constants a_{1,2} account for spatial phase shift. A straightforward linear stability analysis leads to the following coupled dispersion relations

### Table I. Maximal values of the extrema of the transverse cross correlation C_{1/2}(Δx) between the two intensities I_1 and I_2 observed in slices 1 and 2 at thresholds for pattern formation I_c and for dislocation instability I_d. The values observed at Δx = 0 are in bold.

<table>
<thead>
<tr>
<th>C_{1/2}(Δx)</th>
<th>I_c, 2d &gt; 0</th>
<th>I_c, 2d &lt; 0</th>
<th>I_d, 2d &lt; 0</th>
</tr>
</thead>
<tbody>
<tr>
<td>Numerics</td>
<td>−0.85/0.92</td>
<td>−0.60/0.95</td>
<td>−0.50/0.80</td>
</tr>
<tr>
<td>Experiments</td>
<td>−0.65/0.65</td>
<td>−0.56/0.60</td>
<td>−0.42/0.72</td>
</tr>
</tbody>
</table>
where $I_c$ is the intensity threshold for modulational instability. For each sign of $\sigma$, the two sets of solutions are valid as illustrated in Fig. 3 for $\sigma = +10.6$. Moreover, the transverse wave number of the roll patterns as well as the value of the primary instability threshold $I_c$ are independent of the sign of $\sigma$. However, only one of the two sets of solutions possesses the lowest threshold corresponding to the lowest value among all the possible wave numbers $K$.

In the case of Fig. 3, the in-phase solution has the lowest threshold for pattern destabilization ($K = K_c = 0.38I_c^{-1}$). Thus, for $\sigma > 0$ the lowest intensity threshold solution is found for $a_2 = a_1$ and by contrast for $a_2 = -a_1$ if $\sigma < 0$. In other words, the dynamics at threshold is characterized by the generation of stationary roll patterns in each slice with the same wavelength that are spatially in-phase for $\sigma > 0$ and spatially antiphase for $\sigma < 0$. So, while the mechanism responsible for the existence of in-phase and antiphase patterns is the Talbot effect [17], the selection of the in-phase or antiphase solution is directly related to a “lowest threshold principle.” These analytical results confirm the observations of Fig. 2.

Numerical simulations carried out for parameters corresponding to the experimental recordings of Fig. 2 show an excellent qualitative agreement with the experiments as shown in Figs. 4 and 5. They confirm the genuine character of the antiphase dynamics and its dependence with the sign of $2d$. The wavelengths for symmetric values of $2d$ are identical at primary threshold, in agreement with the analytical predictions. The wavelengths in Figs. 4(a) and 4(e) are different because the patterns are calculated for values exceeding the threshold in order to obtain a better contrast. As the variation of the wavelength with the incident power strongly depends on the sign of $\sigma$ (see Fig. 4 in [15]), significantly different wavelengths are observed for positive and negative values of $2d$.

In-phase or antiphase dynamics remain robust even for complex spatiotemporal behaviors such as the dislocation regime observed beyond the secondary threshold $I_d$ (Fig. 5). Indeed, for positive nonlinearity, i.e., the in-phase case, experimental backward and forward dislocation patterns evolve identically. The situation is quite different for antiphase (negative nonlinearity), since the fringes are intertwined. In this dislocation antiphase regime, experimental temporal and spatial evolutions of the fringes are coupled because of the fringe drift motion, thus producing different patterns as, e.g., the connected [Fig. 5(a)] and disconnected [Fig. 5(b)] filaments shown in Fig. 5. The temporal oscillations related to these dislocations do not correspond to a Hopf bifurcation but come from the selection of an unstable wave number due to the Gaussian profile of the pumping beam [15]. The threshold for the dislocation instability is well below the threshold for the Hopf bifurcation so no Turing-Hopf interaction [14,21] occurs. In accordance with these experimental observations, numerics confirm that antiphase is present in the whole domain of parameters, even for complex spatiotemporal regimes such as irregular dislocations [Figs. 5(e)–5(h)]. Although it is difficult to define precisely a global transverse wavelength (except in the central part of the patterns) in this situation, and consequently rigorous antiphase, we observe that the cross correlation $C_{1/2}(\Delta x)$ still remains significantly far from zero. Thus, we may conclude that antiphase spatiotemporal dynamics is a robust property which is not destroyed by additional bifurcations (as a temporal one here) although each of them is accompanied by some symmetry breaking.

To summarize, we have experimentally and theoretically evidenced spatiotemporal antiphase dynamics in spatially extended systems exhibiting a complex spatiotemporal

\[
\begin{align*}
  a_{1,2}(I_{c}^{2}K^2 + i\Omega + 1) &= a_{2,1}2I\chi \sin(\sigma K^2) \\
  a_2 &= \pm a_1, \quad \sin(\sigma K^2) = \pm \frac{1 + K^2}{2\chi I_c},
\end{align*}
\]
behavior, provided that the system is built from coupled similar subsystems. In-phase or antiphase dynamics depends on the sign of the nonlinearity, in our problem, the sign of the coupling distance. When it occurs, spatiotemporal antiphase appears as a robust phenomenon surviving complex behaviors. More sophisticated forms of spatiotemporal antiphase such as “winner takes all” dynamics [3] are expected in experimental devices built from more than two coupled identical subsystems.

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[18] See EPAPS Document No. E-PRLTAO-99-021751 for the auxiliary material. It contains the formula of the cross correlation $C_{1/2}(\Delta t)$ function and two figures depicting the numerical and experimental autocorrelation and cross correlation functions between the two intensities $I_1$ and $I_2$ at thresholds. For more information on EPAPS, see http://www.aip.org/pubservs/epaps.html.
Time Averaged Cross-correlation and Autocorrelation Functions of In-phase and Antiphase Spatiotemporal Dynamics

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The time averaged cross-correlation $C_{1/2}(\Delta x)$ between the two intensities $I_1(x)$ and $I_2(x + \Delta x)$ observed in slices 1 and 2 versus the spatial shift $\Delta x$ reads:

$$C_{1/2}(\Delta x) = \left\langle \frac{\frac{1}{n} \sum_{i=1}^{n} (I_1(x_i) - \bar{I}_1) \times (I_2(x_i + \Delta x) - \bar{I}_2)}{\sqrt{\frac{1}{n} \sum_{i=1}^{n} (I_1(x_i) - \bar{I}_1)^2 \times \frac{1}{n} \sum_{i=1}^{n} (I_2(x_i + \Delta x) - \bar{I}_2)^2}} \right\rangle$$

(1)

where $\bar{I}_1 = \frac{1}{n} \sum_{i=1}^{n} I_1(x_i)$ and $\bar{I}_2 = \frac{1}{n} \sum_{i=1}^{n} I_2(x_i + \Delta x)$. $\langle \rangle$ indicates that the quantity is time averaged. $n$ is the number of sampled points $x_i$ over the studied transverse space. $\Delta x = i \delta x$ is in units of the transverse sampling step $\delta x$. The autocorrelation $C_{1/1}(\Delta x)$ for a single intensity $I_1(x)$ is obtained by replacing $I_2$ with $I_1$ in Eq. (1).

Figure 1: (a-c) Numerical autocorrelation $C_{1/1}(\Delta x)$ and (d-f) cross-correlation $C_{1/2}(\Delta x)$ between the two intensities $I_1$ and $I_2$ observed in slices 1 and 2 at thresholds for pattern formation $I_c$ (a,b,d,e) and for dislocation instability $I_d$ (c,f). (a,d) correspond to positive nonlinearity and (b,c,e,f) to negative nonlinearity. (c,f) Plots in gray correspond to instantaneous values at $t = 0$ for comparison with time averaged ones. Parameters are $w = 140 l_x$, (a,d) $\sigma = 11$, $I = I_c$, (b,e) $\sigma = -10.6$, $I = I_c$, (c,f) $\sigma = -10.6$, $I = I_d = 1.14I_c$. 

Figure 2: (a-c) Experimental autocorrelation $C_{1/1}(\Delta x)$ and (d-f) cross-correlation $C_{1/2}(\Delta x)$ between the two intensities $I_1$ and $I_2$ observed in slices 1 and 2 at thresholds for pattern formation $I_c$ (a,b,d,e) and for dislocation instability $I_d$ (c,f). (a,d) correspond to positive nonlinearity and (b,c,e,f) to negative nonlinearity. (c,f) Plots in gray correspond to instantaneous values at $t = 0$ for comparison with time averaged ones. Parameters are $w = 1400 \ \mu m$, (a,d) $d = 13 \ mm \ (\sigma = 11)$, $I = I_c$, (b,e) $d = -12.5 \ mm \ (\sigma = -10.6)$, $I = I_c$, (c,f) $d = -12.5 \ mm \ (\sigma = -10.6)$, $I = I_d = 1.35I_c$. 