Elements of quantum gases

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This course gives an introduction into the physics of trapped atomic gases with emphasis on its foundation in elementary statistical physics and quantum mechanics.

The starting point is the quasi-classical gas at low densities. Emphasis is put on the presence of a trapping potential. The density and momentum distributions are derived along with some thermodynamic and kinetic properties. All these aspects meet in a discussion of evaporative cooling.

Introducing short-range interatomic interactions we find that in thermal equilibrium low density gases exist only at high temperature. For ultra-cold gases this leads to the concepts of quasi-equilibrium and metastability. Depending on the density the gas may behave collisionless or hydrodynamic. The limitations of the classical description are discussed by introducing the quantum resolution limit in the classical phase space. The notion of a quantum gas appears by comparing the thermal de Broglie wavelength with characteristic length scales of the gas: the range of the interatomic interaction, the interatomic spacing and the size of a gas cloud. Asking the question 'what makes a gas into a quantum gas?' we examine the role of state quantization, quantum collisions, quantum statistical effects and the energy of the ground state.

Then we turn to binary interactions in a gas at temperatures where the thermal wavelength exceeds the range of the interaction potential. The underlying idea is that a lot can be learned about nearly-ideal quantum gases by considering no more than two atoms confined to a finite volume. The discussion is fully quantum mechanical. It is restricted to elastic interactions and short-range potentials in the zero-energy limit. Particular attention is paid to the analytically solvable cases. The central quantities are the asymptotic phase shift and the s-wave scattering length. The relation between bound states and scattering length is discussed. It is shown how the phase shift in combination with the boundary condition of the confinement volume suffices to calculate the energy of interaction between the atoms. We analyze how the permutation operator affects the pair wavefunction and the interaction energy for the case of bosons and fermions.

The course ends with a discussion of kinetic quantities, in particular the scattering amplitude and the differential and total cross section, including their relation with the phase shift. Aside from the zero-energy limit, we discuss the energy dependence of low-energy scattering, including the Ramsauer-Townsend effect, s-wave resonances and shape resonances. Including coupling to a second channel (the ‘closed’ channel) gives rise to Feshbach resonances, which enable in-situ variation of the scattering length. In this context the concepts of background scattering length and wide versus narrow resonances are introduced.